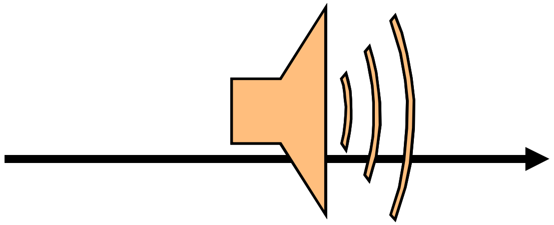
CHAPTER 3

ELECTROMECHANICAL ENERGY CONVERSION

Electrical energy is the most popular form of energy, because:

1. it can be transmitted easily for long distance, at high efficiency and reasonable cost.

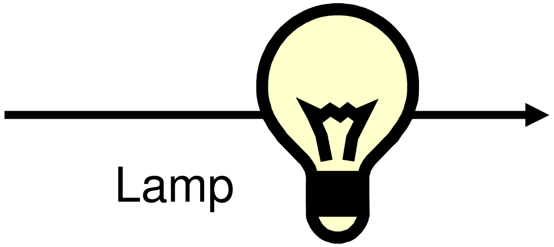
2. It can be converted easily to other forms of energy such as sound, light, heat or mechanical energy.



Electrical energy

Sound energy

Loud speaker



Electrical energy

Light energy

Lamp

Electrical energy

Heat energy

Kettle

Electromechanical energy conversion device:

converts electrical energy into mechanical energy

or

converts mechanical energy into electrical energy.

An **electrical machine** is a device that can convert either mechanical energy to electrical energy or electrical energy to mechanical energy. When such a device is used to convert mechanical energy to electrical energy, it is called a *generator.* When it converts electrical energy to mechanical energy, it is called a *motor.* Since any given electrical machine can convert power in either direction, any machine can be used as either a generator or a motor. Almost all practical motors and generators convert energy from one form to another through the action of a magnetic field, and only machines using magnetic fields to perform such conversions are considered in this book.

The *transformer* is an electrical device that is closely related to electrical machines. It converts ac electrical energy at one voltage level to ac electrical energy at another voltage level. Since transformers operate on the same principles as generators and motors, depending on the action of a magnetic field to accomplish the change in voltage level, they are usually studied together with generators and motors.

These three types of electric devices are ubiquitous in modern daily life. Electric motors in the home run refrigerators, freezers, vacuum cleaners, blenders, air conditioners, fans, and many similar appliances. In the workplace, motors provide the motive power for almost all tools. Of course, generators are necessary to supply the power used by all these motors.

As previously stated, magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers. Four basic principles describe how magnetic fields are used in these devices:

1. A current-carrying wire produces a magnetic field in the area around it.
2. A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil. (This is the basis of *transformer action*.)
3. A current-carrying wire in the presence of a magnetic field has a force induced on it. (This is the basis of *motor action*.)
4. A moving wire in the presence of a magnetic field has a voltage induced in it. (This is the basis of *generator action*.)

In pure magnetic-field systems, the situation is somewhat more complex. Here the force

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \tag{3.3}$$

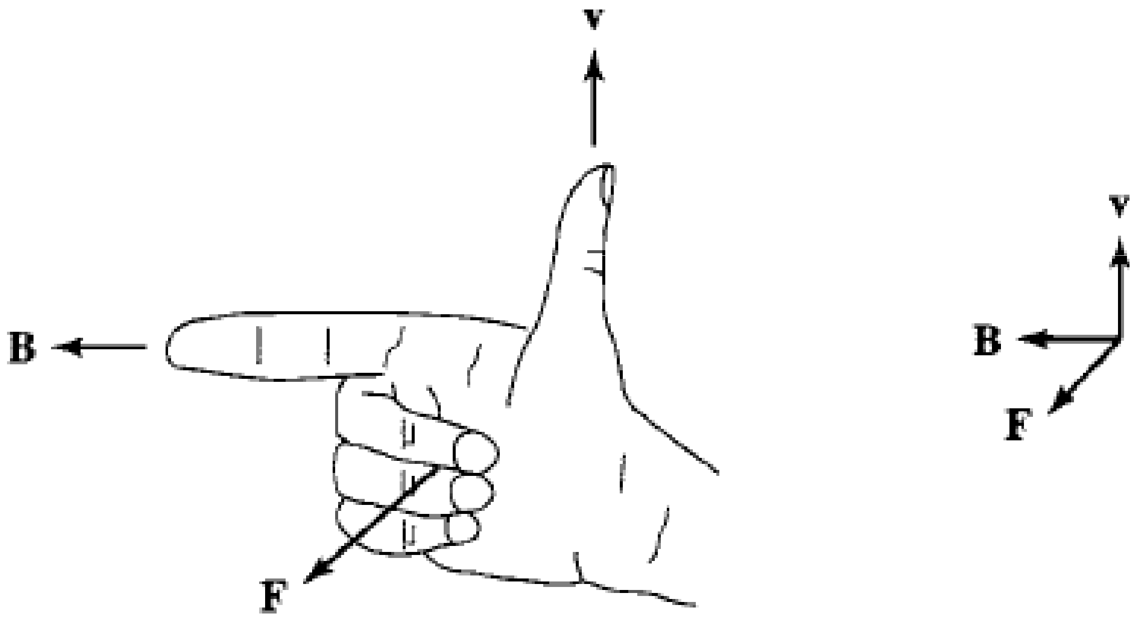


Figure 3.1 Right-hand rule for determining the direction magnetic-field component of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

1.6 PRODUCTION OF INDUCED FORCE ON A WIRE

A second major effect of a magnetic field on its surroundings is that it induces a force on a current-carrying wire within the field. The basic concept involved is illustrated in Figure 1–16. The figure shows a conductor present in a uniform magnetic field of flux density **B**, pointing into the page. The conductor itself is *l* meters long and contains a current of *i* amperes. The force induced on the conductor is given by

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) \tag{1–43}$$

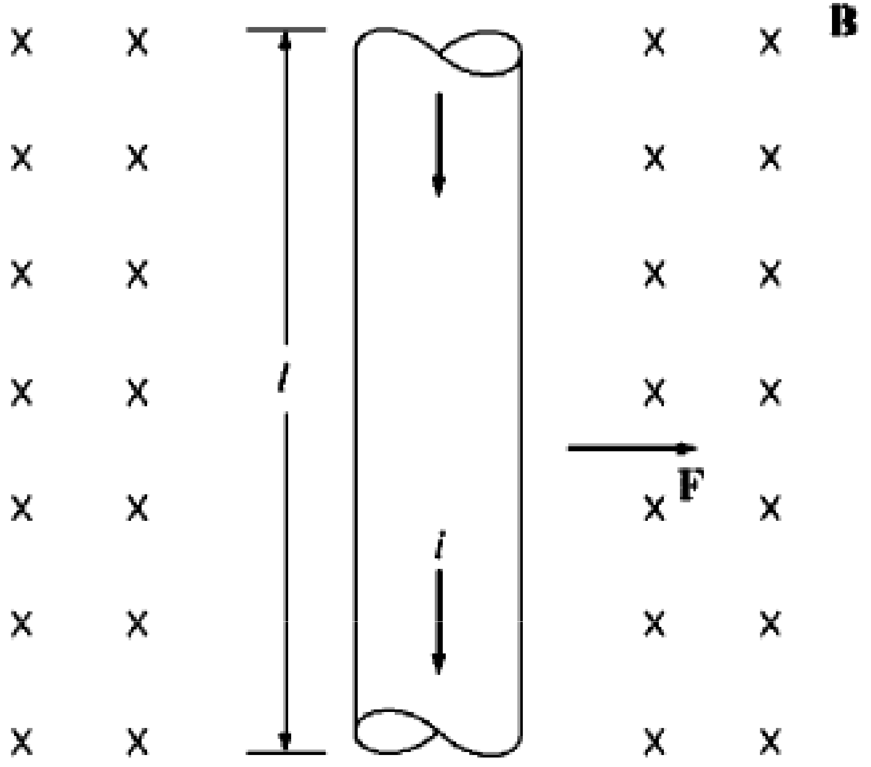


FIGURE 1–16
A current-carrying wire in the presence of a magnetic field.

where

i = magnitude of current in wire

$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow

$\mathbf{B}$ = magnetic flux density vector

The direction of the force is given by the right-hand rule: If the index finger of the right hand points in the direction of the vector **l** and the middle finger points in the direction of the flux density vector **B**, then the thumb points in the direction of the resultant force on the wire. The magnitude of the force is given by the equation

$$F = ilB \sin \theta \tag{1–44}$$

where θ is the angle between the wire and the flux density vector.

1.7 INDUCED VOLTAGE ON A CONDUCTOR MOVING IN A MAGNETIC FIELD

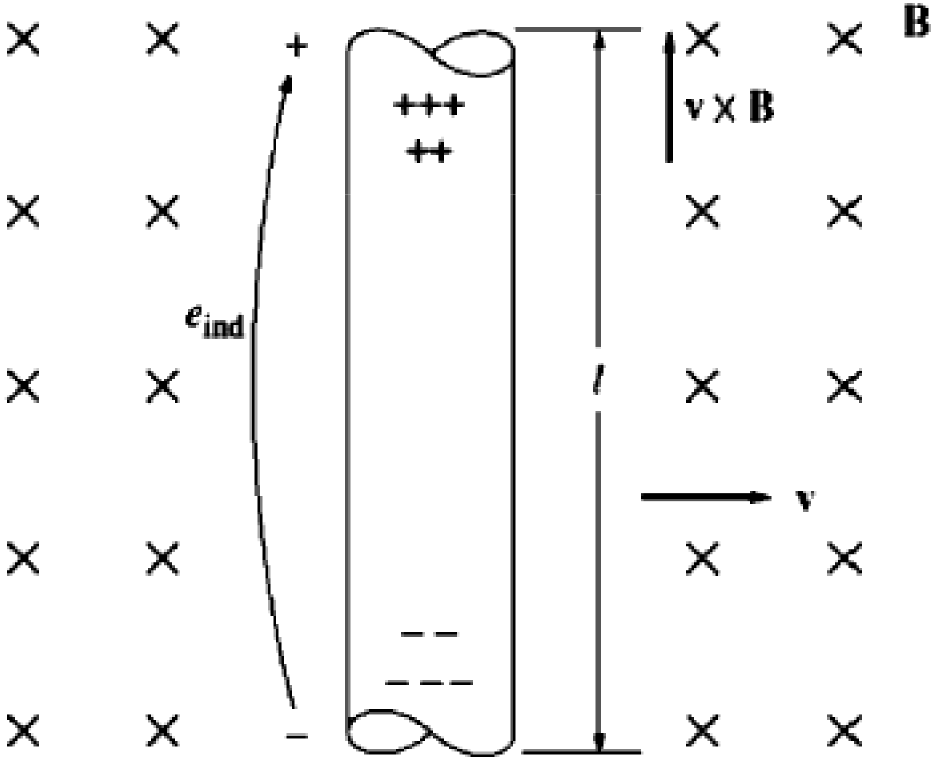
There is a third major way in which a magnetic field interacts with its surroundings. If a wire with the proper orientation moves through a magnetic field, a voltage is induced in it. This idea is shown in Figure 1–17. The voltage induced in the wire is given by

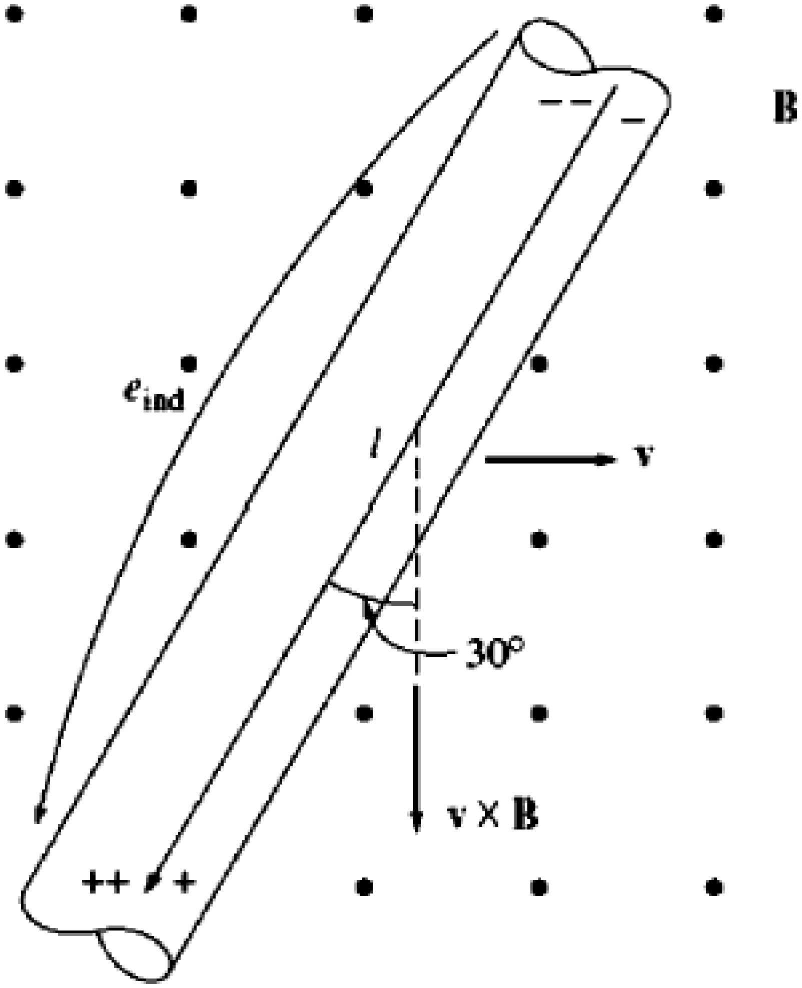
$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

where

$\mathbf{v}$ = velocity of the wire
$\mathbf{B}$ = magnetic flux density vector
$\mathbf{l}$ = length of conductor in the magnetic field

Vector **l** points along the direction of the wire toward the end making the smallest angle with respect to the vector **v** × **B**. The voltage in the wire will be built up so that the positive end is in the direction of the vector **v** × **B**. The following examples illustrate this concept.




B
e_{ind}
l
v
30°
v × B
++ +
– – –

There are various electromechanical conversion devices may categorized as under:

a. Small motion

- telephone receivers, loud speakers, microphones

b. Limited mechanical motion

- electromagnets, relays, moving-iron instruments, moving-coil instruments, actuators

c. Continuous energy conversion

- motors, generators

Principle of Energy Conversion

According to the ***principle of conservation of energy***, energy can neither be created nor destroyed,

it can merely be converted from one form into another.

The total energy in a system is therefore constant.

Energy conversion in electromechanical system

In an energy conversion device, out of the total input energy, some energy is converted into the required form, some energy is stored and the rest is dissipated.

It is possible to write an equation describing energy conversion in electromechanical system:

Electrical energy from source = Mechanical energy to load + Increase of field energy + Energy converted to heat (losses) —— (3.1)

Electrical energy from source = Mechanical energy to load + Increase of field energy + Energy converted to heat (losses) ③.①

The last term on the right-hand side of Eq. 3.1 (the losses) may be divided into three parts:

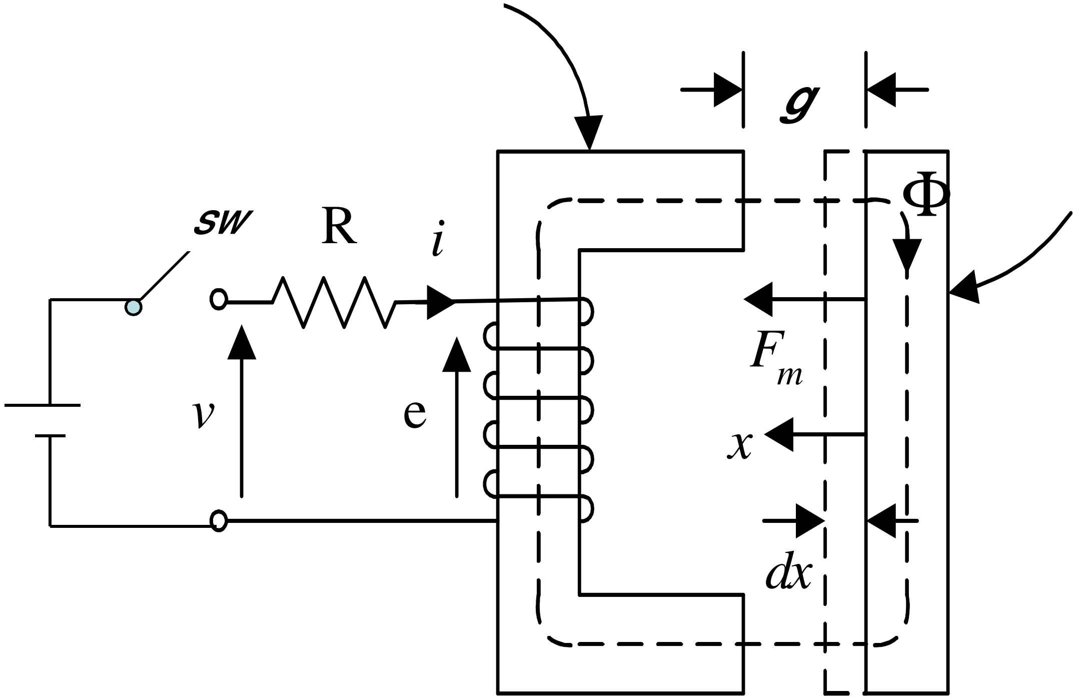
Energy converted to heat (losses) = Resistance losses + Friction and windage losses + Field losses (3.2)

Then substitution from Eq. 3.2 in Eq. 3.1 yields

Electrical energy from source minus resistance losses = Mechanical energy to load plus friction and windage losses + Increase of magnetic coupling field energy plus core losses (3.3)

Now consider an electromechanical system (actuator) illustrated in Fig. 3.1.

Fixed steel core



Moveable steel armature

Figure 3.1

At any instant, the emf e induced in the coil by the change in the flux linkage λ is

$$e = \frac{d\lambda}{dt} \qquad \text{volt} \tag{3.4}$$

Consider now a differential time interval ***dt***; during which the current in the coil is changing and the armature is moving.

Therefore, the differential energy transferred in time ***dt*** from the electric source to the coupling field is given by the energy output of the source minus the resistance loss:

$$dW_e = vidt - Ri^2dt$$

$$= (v - Ri)idt$$

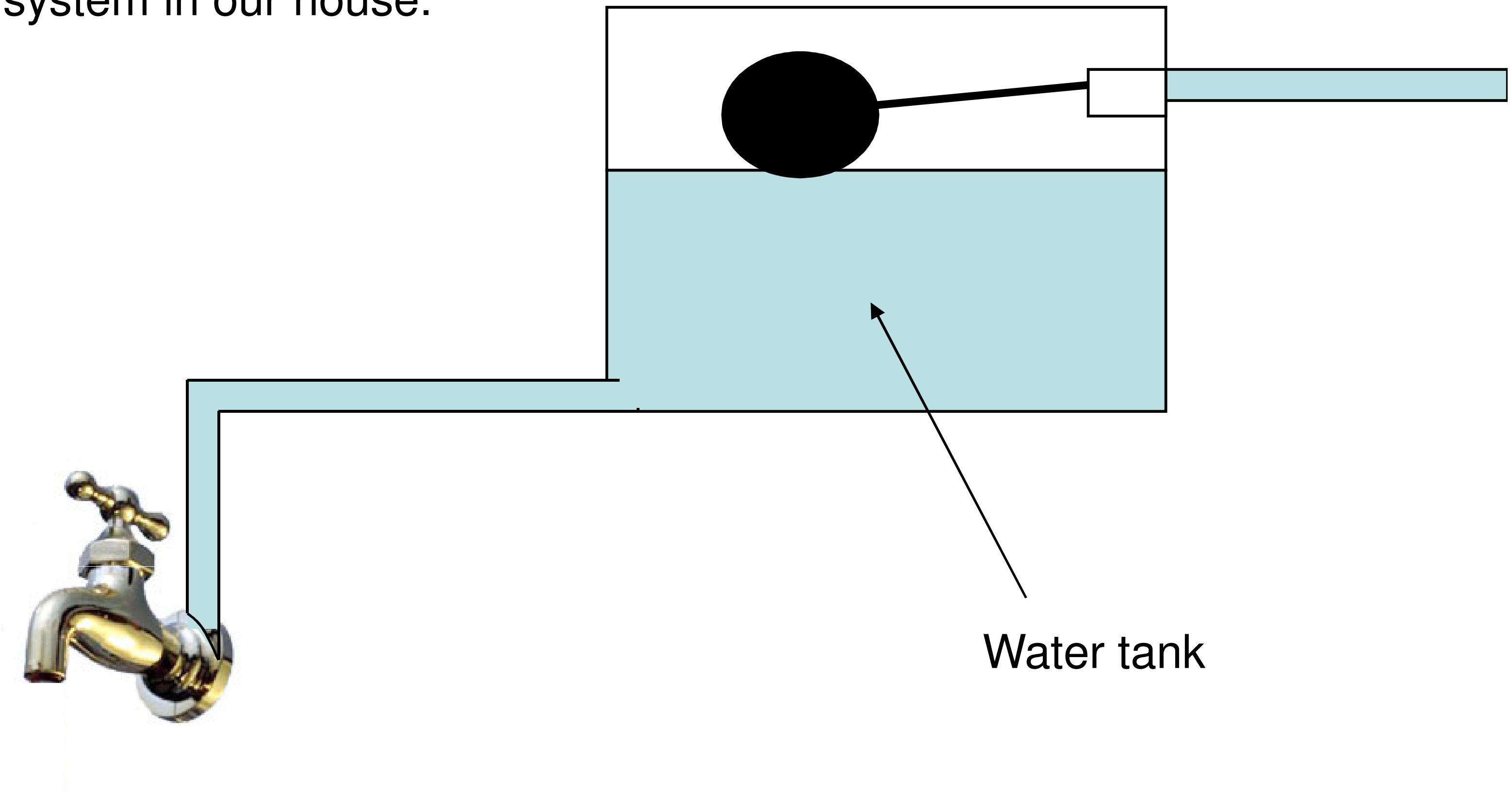
$$= eidt$$

$$\Rightarrow \qquad dW_e = eidt \qquad \text{Joule} \tag{3.5}$$

The coupling field forms an energy storage to which energy supplied by the electric system. At the same time, energy is released from the coupling field to the mechanical system.

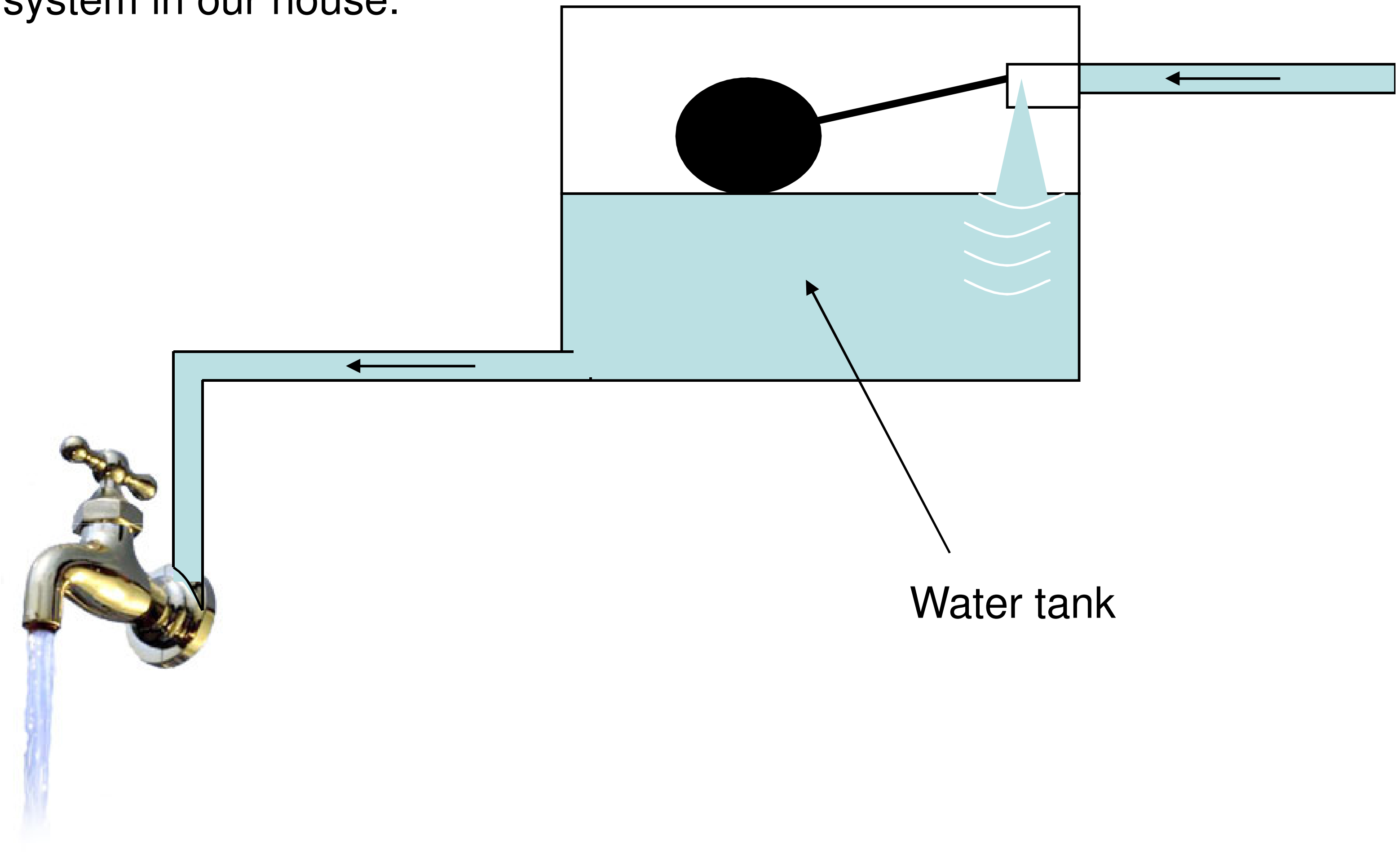
The rate of release energy is not necessarily equal at any instant to the rate of supply of energy to the field, so that the amount of energy stored in the coupling field may vary.

It’s like a pipe system in our house.



The water out from the tap will make water flow into the storage tank from the supply.

It’s like a pipe system in our house.



The water out from the tap will make water flow into the storage tank from the supply.

In time ***dt***, let ***dW*** be the energy supplied to the field and either stored or dissipated. Let dW_m be the energy converted to mechanical form, useful or as loss, in the same time, ***dt***.

Then, by the ***principle of conservation of energy***, the following equation may be written for the field:

$$dW_e = dW_m + dW_f \qquad (3.6)$$

Field Energy

To obtain an expression for for dW_f of Eq. 3.6 in terms of the system variables, it is first necessary to find an expression for the energy stored in the magnetic field for any position of the armature. The armature will therefore be clamped at some value of air-gap length g so that no mechanical output can be produced.

$$dW_m = 0 \qquad \text{(3.6)}$$

Field Energy (continue…..)

If switch SW in Fig. 3.1 is now closed, the current will rise to a value ***v/R***, and the flux will be established in the magnetic system. Let the relationship between coil flux linkage λ and the current ***i*** for the chosen air-gap length be that shown in Fig. 3.2

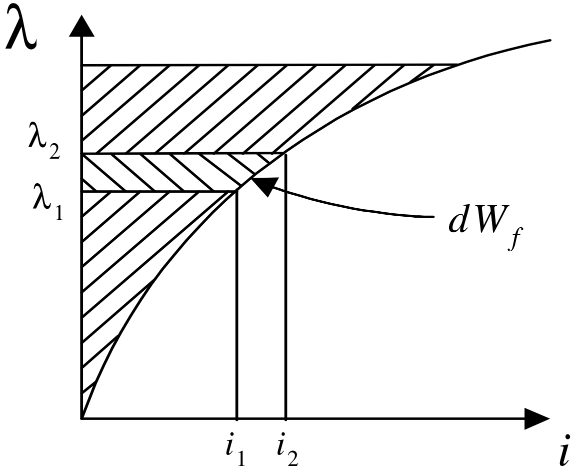


Fig. 3.2

Field Energy (continue…..)

Since core loss is being neglected, this will be a single-valued curve passing through the origin. In the absence of any mechanical output energy, all of the electric input energy must be stored in the magnetic field:

$$dW_e = dW_f \tag{3.8}$$

Substitution from Eqs. 3.4 and 3.8 in Eq. 3.5 yields

$$dW_f = dW_e = i.e\,dt = i\,d\lambda \quad \text{J} \tag{3.9}$$

Field Energy (continue…..)

If now v is changed, resulting in a change in current from i_1 to i_2, there will be a corresponding change in flux linkage from λ_1 to λ_2.

The increase in energy stored during the transition between these two states is

$$dW_f = \int_{\lambda_1}^{\lambda_2} i\,d\lambda \qquad \text{J} \tag{3.10}$$

The area is shown in Fig 3.2. When the flux linkage is increased from zero to λ, the total energy stored in the field is

Field Energy (continue…..)

$$W_f = \int_0^{\lambda} i d\lambda \quad \text{J}$$ ③.11

This integral represents the area between the λ–i characteristic and the λ–axis, the entire shaded area of Fig. 3.2.

If it is assumed that there is no leakage flux, so that all flux Φ in the magnetic system links all N turns of the coil, then

$$\lambda = N\Phi \quad \text{Wb}$$ (3.12)

Field Energy (continue…..)

From Eqs. 3.9 and 3.12,

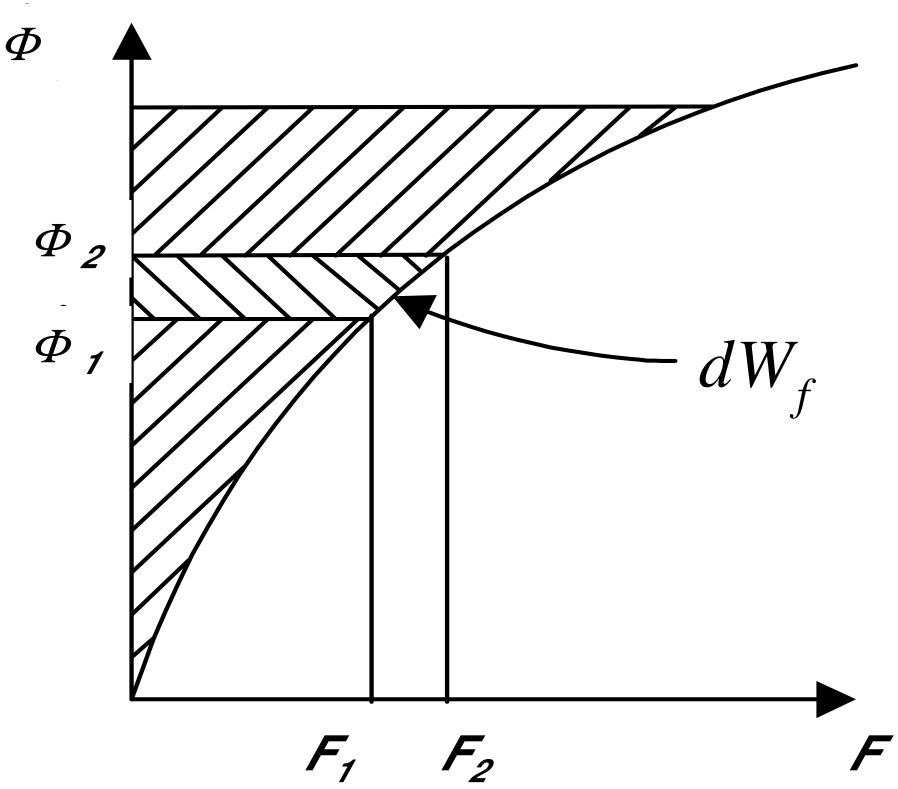
$$dW_f = i d\lambda = N i d\Phi = \mathscr{F} d\Phi \quad \text{J} \qquad \text{③.13}$$

where

$$\mathscr{F} = Ni \quad \text{A} \qquad \text{③.14}$$

$\mathscr{F}$ is mmf (magneto-motive force)

The characteristic of Fig. 3.3 can also represent the relationship between Φ and $\mathscr{F}$.


Φ
Φ 2
Φ 1
dW_f
F1
F2
F

Field Energy (continue…..)

If the reluctance of the air gap forms a large part of the total reluctance of the magnetic system, then that of the steel may be neglected and the $\lambda - i$ characteristic becomes the straight line through the origin shown in Fig. 3.3. For this system,

$$\lambda = Li \quad \text{Wb} \qquad (3.15)$$

Where L is the inductance of the coil. Substitution in Eq. 3.11 gives the energy W_f in several useful forms:

$$W_f = \int_0^{\lambda} \frac{\lambda}{L} d\lambda = \frac{\lambda^2}{2L} = \frac{Li^2}{2} = \frac{i\lambda}{2} \quad \text{J} \qquad (3.16)$$

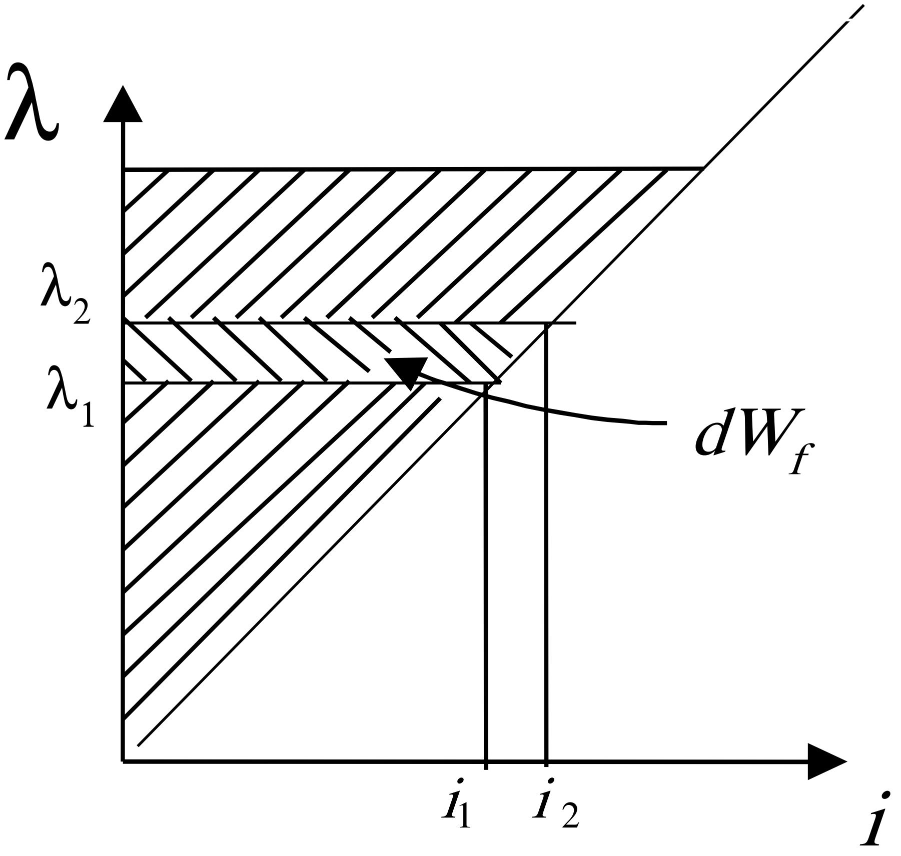


Fig. 3.3

Field Energy (continue…..)

If the reluctance of the magnetic system (that is, of the air gap) as seen from the coil is $\boldsymbol{S}$, then $\mathscr{F} = \boldsymbol{S}\,\Phi$, and from Eq. 3.13,

$$W_f = \int_0^{\phi} \mathscr{F}\, d\lambda = \frac{S\phi^2}{2} = \frac{\mathscr{F}^2}{2S} \quad \text{J} \tag{3.17}$$

If $\boldsymbol{A}$ is the cross-section area of the core and $\boldsymbol{l = 2g}$ is the total length of air gap in a flux path, then from Eq. 3.16,

$$W_f = \frac{i\lambda}{2} = \frac{F\phi}{2} = \frac{1}{2} HBlA \quad \text{J} \tag{3.18}$$

Field Energy (continue…..)

Where B is the flux density in the air gaps. Since $B/H = \mu_0$ and lA is the total gap volume, it follows from Eq. 3.18 that the energy density in the air gaps is

$$w_f = \frac{W_f}{lA} = \frac{1}{2}BH = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{J/m}^3 \qquad (3.19)$$

Equations 3.16, 3.17 and 3.19 represent three different ways of expressing the field energy.

Example 3.1 The core and armature dimensions of the actuator of Fig. 3.1 are shown in Fig. 3.4. Both parts are made of mild steel, whose magnetization curve is given in Fig. 3.5. Given l_a = 160 mm, l_b = 80 mm. The coil has 2000 turns. Leakage flux and fringing may be neglected. The armature is fixed, so that the length of the air gas, l_g = 9 mm, and a direct current is passed through the coil, producing a flux density of 0.8 T in the air gap.

a) Determine the required coil current.

b) Determine the energy stored in the air gap.

c) Determine the energy stored in the steel.

d) Determine the total field energy.

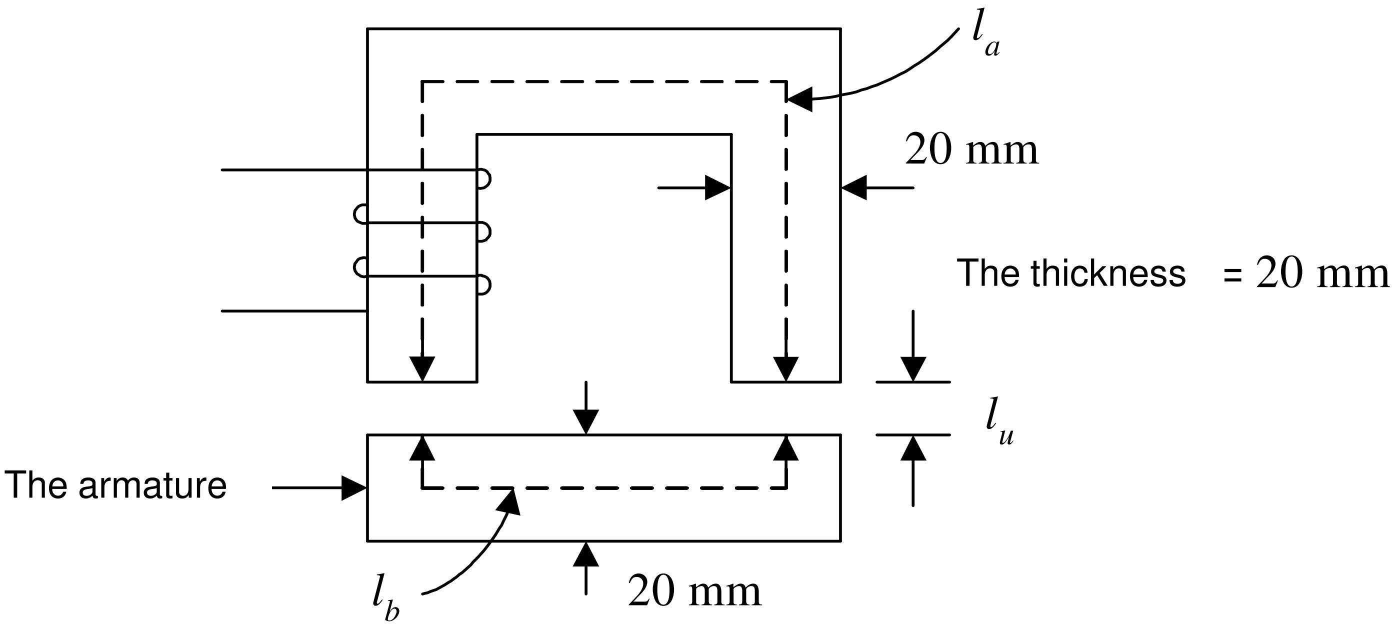


Fig. 3.4

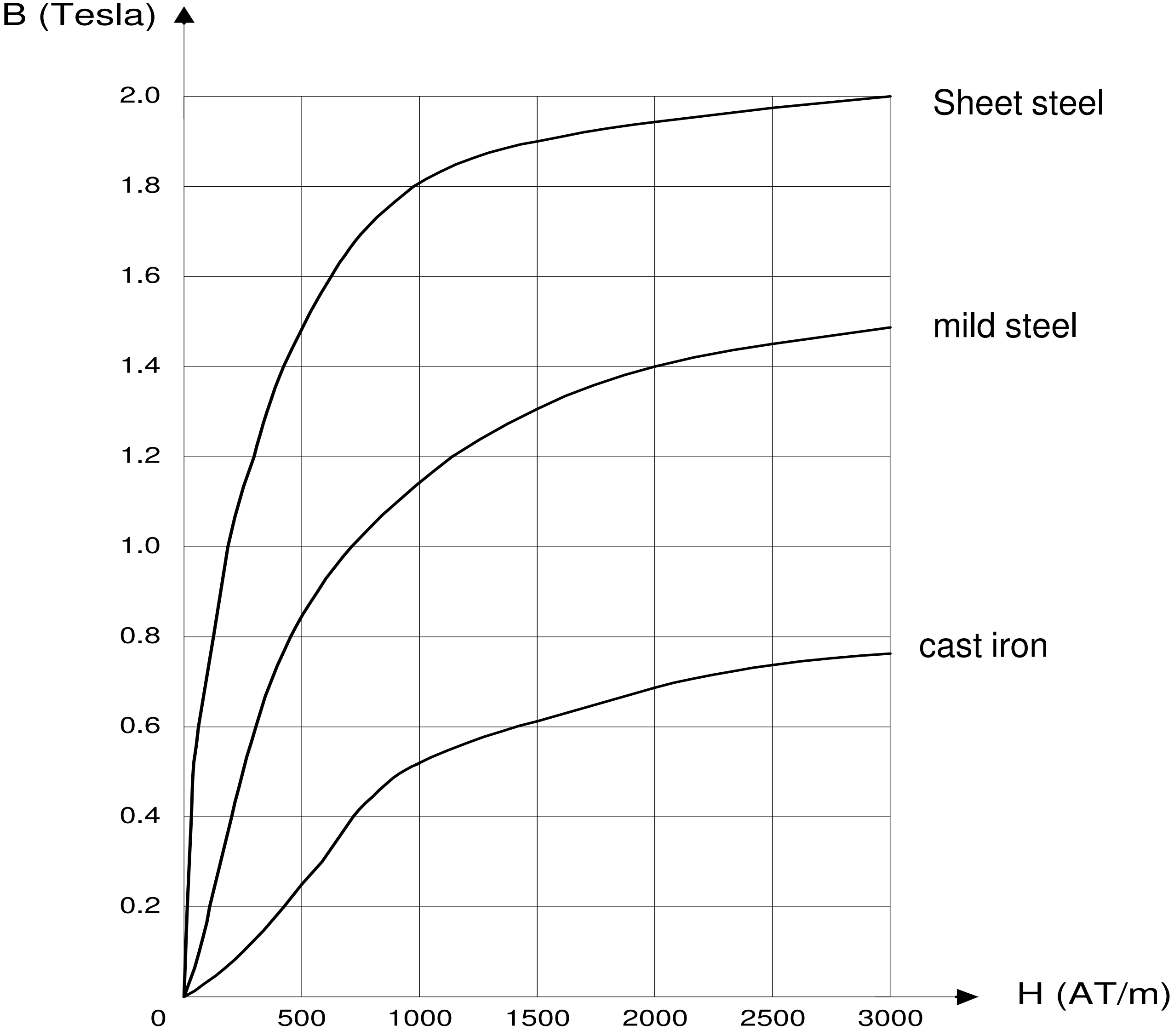

B (Tesla)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
500
1000
1500
2000
2500
3000
H (AT/m)
Sheet steel
mild steel
cast iron

Fig. 3.5

Solution

(a) Area, $A = (20 \times 10^{-3})(20 \times 10^{-3}) = 4 \times 10^{-4}\ \text{m}^2$.

$$Ni = H_t l_t + H_u l_u$$

$l_t = 160 + 80 = 240\ \text{mm} = 240 \times 10^{-3}\ \text{m}$
$l_u = 2 \times 9\ \text{mm} = 18\ \text{mm} = 18 \times 10^{-3}\ \text{m}$

Given $B_u = 0.8\ \text{T}$
$B_u = B_t = 0.8\ \text{T}$

From Fig. 3.6, magnetic field intensity in the steel is,
$H_t = 450\ \text{A/m}$

For the air gaps

$$H_u = \frac{B_u}{\mu_o} = \frac{0.8}{4\pi\times10^{-7}} = 636.62\times10^{3} \quad \text{A/m}$$

$$i = \frac{(450)(240\times10^{-3}) + (636.62\times10^{3})(18\times10^{-3})}{2000}$$

$$= \frac{11567.16}{2000} = 5.78 \quad \text{A}$$

(b) Energy density in the air gaps is

$$w_{fu} = \frac{B^2}{2(4\pi\times10^{-7})} = 254.65\times10^{3} \quad \text{J/m}^3$$

$$
\begin{aligned}
\text{Volume of air gaps} &= \text{length of air gaps} \times \text{area of air gaps} \\
&= 0.018 \times 0.02 \times 0.02 \\
&= 7.2 \times 10^{-6}\ \text{m}^3
\end{aligned}
$$

Energy stored in the air gaps,

$$
\begin{aligned}
W_{fu} &= \text{the volume of air gaps} \times w_{fu} \\
&= (7.2 \times 10^{-6}) \times 254.65 \times 10^3 \\
&= 1.834\ \text{Joule.}
\end{aligned}
$$

(c) Energy density in the steel,

$$
w_{ft} = \int_0^{0.8} H dB
$$

Energy density in the steel is given by the area enclosed between the characteristic and the ***B*** axis in Fig. 3.6 up to value of 0.8 T.

$$w_{ft} \cong \tfrac{1}{2} \times 0.8 \times 450 = 180 \text{ J/m}^3 \text{ (straight-line approximation)}$$

Volume of steel= length of steel × area of steel

$$= (240 \times 10^{-3}) \times (0.02 \times 0.02)$$

$$= 9.6 \times 10^{-5} \text{ m}^3$$

∴Energy stored in the steel,

$$W_{ft} = 9.6 \times 10^{-5} \times 180 = 0.01728 \text{ Joule}$$

(d) Total field energy,

$$W_f = W_{ft} + W_{fu}$$

$$= 0.01728 + 1.834$$

$$= 1.851 \text{ Joule.}$$

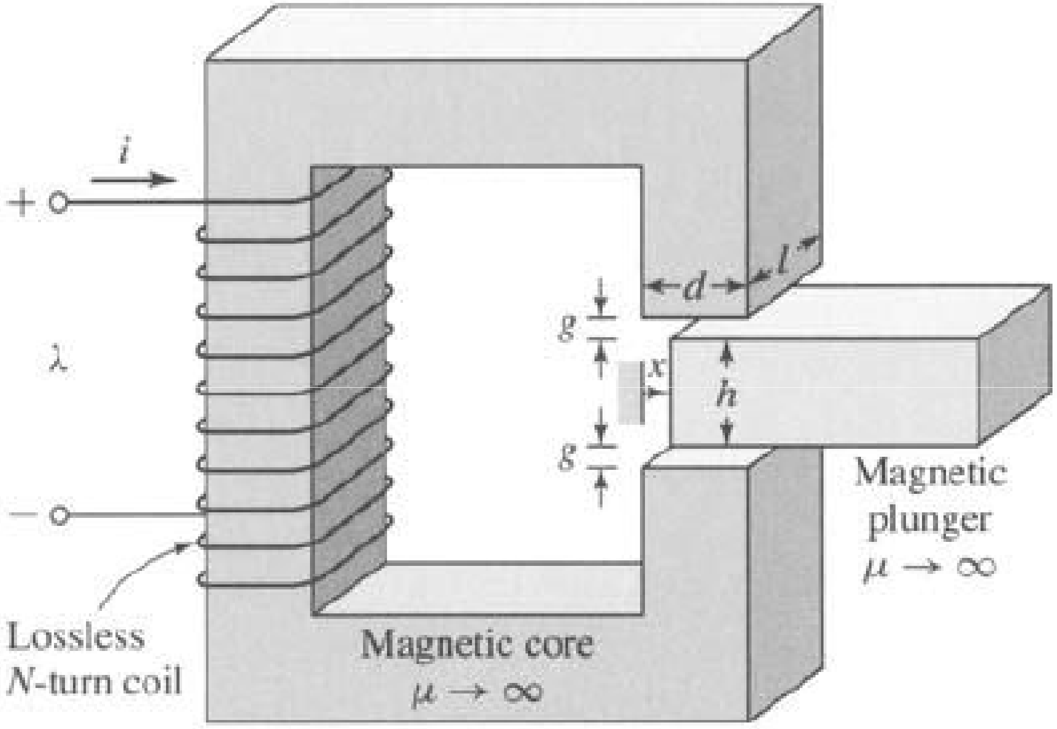
The proportion of field energy stored in the steel is, therefore, seen to be negligibly.

Example

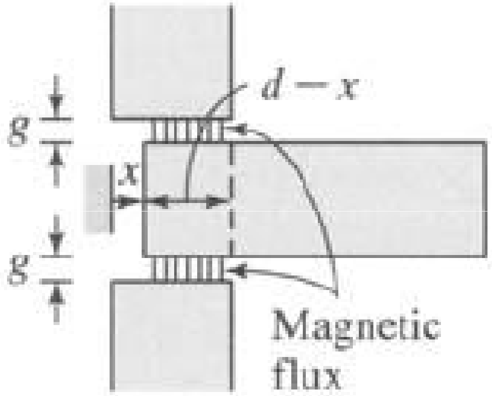
The relay shown in Fig. 3.6a is made from infinitely-permeable magnetic material with a movable plunger, also of infinitely-permeable material. The height of the plunger is much greater than the air-gap length ($h \gg g$). Calculate the magnetic stored energy W_{fld} as a function of plunger position ($0 < x < d$) for $N = 1000$ turns, $g = 2.0$ mm, $d = 0.15$ m, $l = 0.1$ m, and $i = 10$ A.

$$W_{\text{fld}} = \frac{1}{2} L(x) i^2$$

$$L(x) = \frac{\mu_0 N^2 A_{\text{gap}}}{2g}$$


i
+
λ
−
g
x
d
l
h
g
Magnetic plunger
μ → ∞
Lossless
N-turn coil
Magnetic core
μ → ∞

(a)


d − x
g
x
g
Magnetic
flux

where A_{gap} is the gap cross-sectional area. From Fig. 3.6b, A_{gap} can be seen to be

$$A_{\text{gap}} = l(d - x) = ld\left(1 - \frac{x}{d}\right)$$

Thus

$$L(x) = \frac{\mu_0 N^2 ld(1 - x/d)}{2g}$$

and

$$\begin{aligned} W_{\text{fld}} &= \frac{1}{2}\frac{N^2\mu_0 ld(1 - x/d)}{2g}\,i^2 \\ &= \frac{1}{2}\frac{(1000^2)(4\pi \times 10^{-7})(0.1)(0.15)}{2(0.002)} \times 10^2\left(1 - \frac{x}{d}\right) \\ &= 236\left(1 - \frac{x}{d}\right)\text{ J} \end{aligned}$$

EXAMPLE

The relay of Fig. 3.6 is modified in such a fashion that the air gaps surrounding the plunger are no longer uniform. The top air gap length is increased to $g_{\mathrm{top}} = 3.5$ mm and that of the bottom gap is increased to $g_{\mathrm{bot}} = 2.5$ mm. The number of turns is increased to $N = 1500$. Calculate the stored energy as a function of plunger position ($0 < x < d$) for a current of $i = 5$ A.

Solution

$$L = \frac{N^2}{\mathcal{R}_{\mathrm{tot}}}$$

$$\mathcal{R}_1 = \frac{g_1}{\mu_0 A_1} \qquad \mathcal{R}_2 = \frac{g_2}{\mu_0 A_2}$$

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2$$

- $R_t = \frac{1}{\mu_0 A}(g_1 + g_2)$
- $g_1 + g_2 = 6x10^{-3}\ m$

$$A_{\text{gap}} = l(d - x) = ld\left(1 - \frac{x}{d}\right)$$

$$W_{\text{fld}} = \frac{1}{2}L(x)i^2$$

$$W_{\text{fld}} = 88.5\left(1 - \frac{x}{d}\right)\ \text{J}$$

Coenergy

Coenergy, ***W$_f$'*** is the area enclosed between the λ-***i*** characteristic and the ***i*** axis of Fig.3.2.

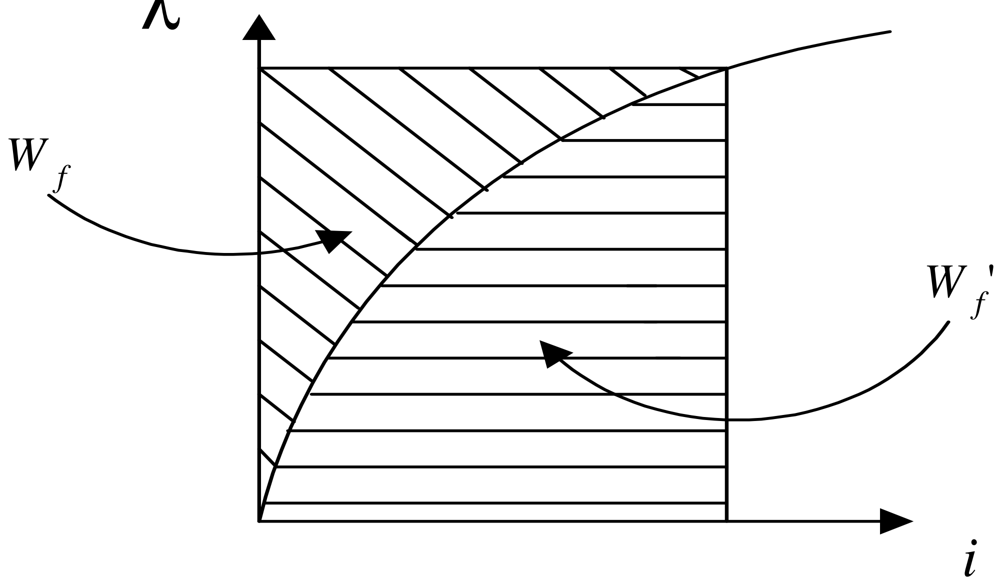


Fig. 3.6 Field energy and coenergy

For linear λ-***i*** characteristic, ***W$_f$'*** = ***W$_f$***
For nonlinear λ-***i*** characteristic, ***W$_f$'*** > ***W$_f$***

Mechanical Energy in a Linear System

It will be assumed that the armature of the actuator in Fig. 3.1 may move from position x_1 to position x_2, as a result, the length of air gaps is reduced. The $\lambda - i$ characteristics for the two extreme positions of the armature may be assumed to be the two straight lines (linear).

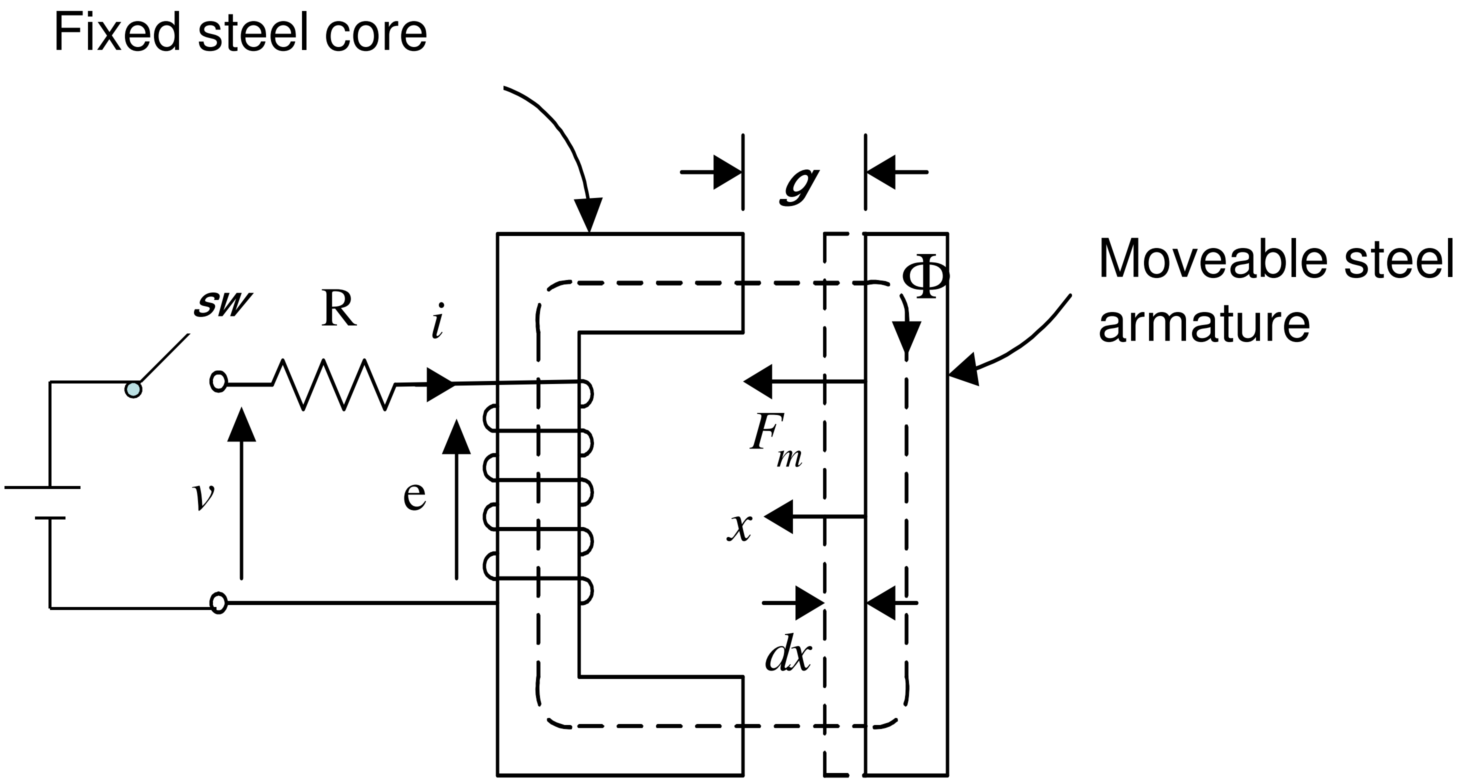
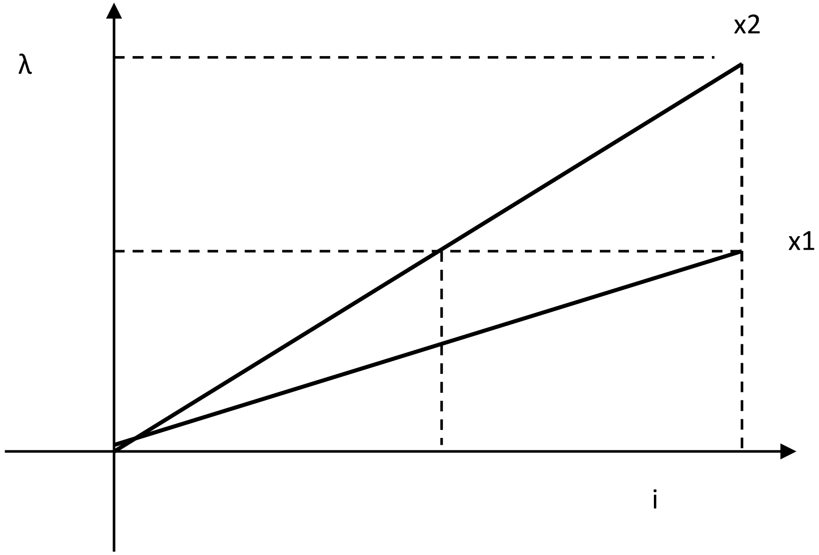


Figure 3.1


x2
λ
x1
i

Mechanical Energy in a Linear System

Consider a very slow armature displacement. It may assumed that it takes place at essentially constant current as illustrated in Fig. 3.7 (as $d\lambda/dt$ is negligible). The operational point has changed from a to b.

At the moment of armature movement,

$$\Delta W_e = \int eidt = \int_{\lambda_1}^{\lambda_2} id\lambda = i_o(\lambda_2 - \lambda_1) \tag{3.20}$$

The change of field energy,

$$\begin{aligned}\Delta W_f &= \tfrac{1}{2} i_o \lambda_2 - \tfrac{1}{2} i_o \lambda_1 \\ &= \tfrac{1}{2} i_o(\lambda_2 - \lambda_1) \end{aligned} \tag{3.21}$$

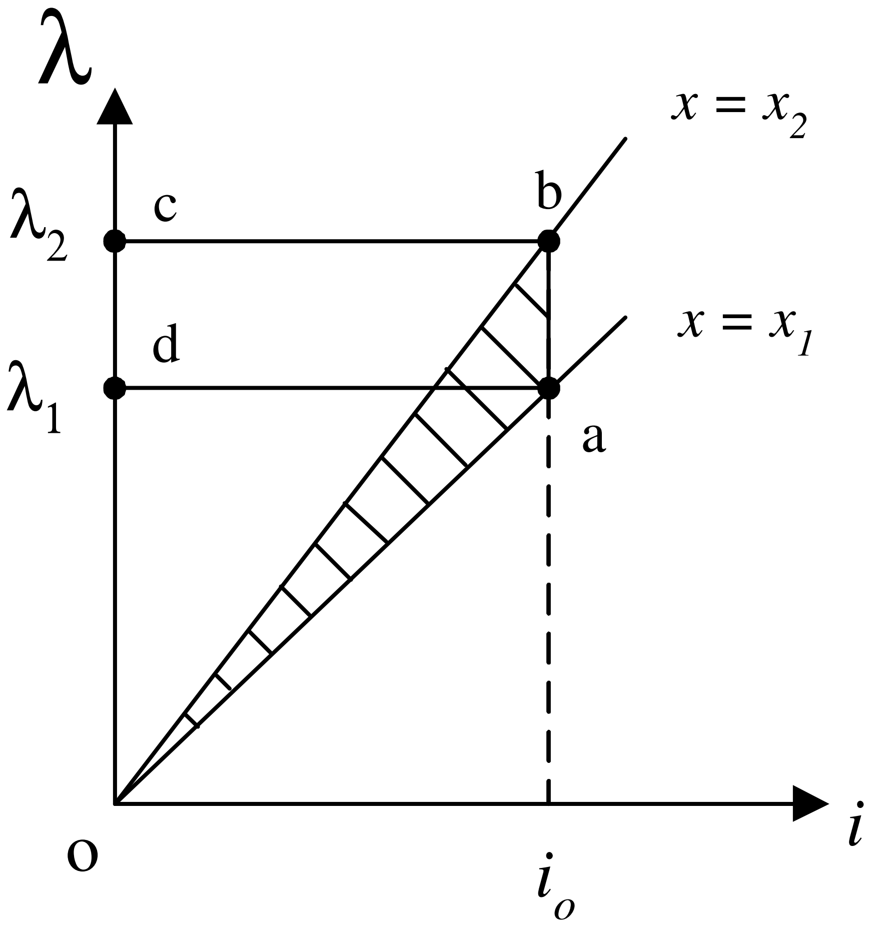


Fig. 3.7 Current is fixed

Mechanical Energy in a Linear System

From Eq. (3.6),

$$\Delta W_e = \Delta W_m + \Delta W_f$$

$$\Delta W_m = \Delta W_e - \Delta W_f$$

$$\therefore \quad = i_o(\lambda_2 - \lambda_1) - \tfrac{1}{2} i_o(\lambda_2 - \lambda_1)$$

$$= \tfrac{1}{2} i_o(\lambda_2 - \lambda_1)$$

$$= \Delta W_f$$

$= \Delta W'_f$ = the change of coenergy

Mechanical Energy in a Linear System

For small change of ***x*** or ***dx***,

$$\because \qquad dW_m = dW_f'$$

$$\Rightarrow \qquad F_m dx = dW_f' \tag{3.21}$$

where

$$\boldsymbol{dW_m = F_m dx}$$

$\boldsymbol{F_m}$ = mechanical force on moving part (armature)

Mechanical Energy in a Linear System

Eq. 3.21 can be written as,

$$\boxed{F_m = \frac{\partial W_f'}{\partial x}(i, x) \Bigg|_{i = \text{constant}} \qquad \text{N}} \tag{3.22}$$

Eq. 3.22 is partial differential since W_f is function of more than one variable.

Consider now a very rapid differential armature displacement dx. It may be assumed that it takes place at essentially constant flux linkage λ_o as illustrated in Fig. 3.8. At the instant, the current is changed from i_1 to i_2 , where $i_1 > i_2$.

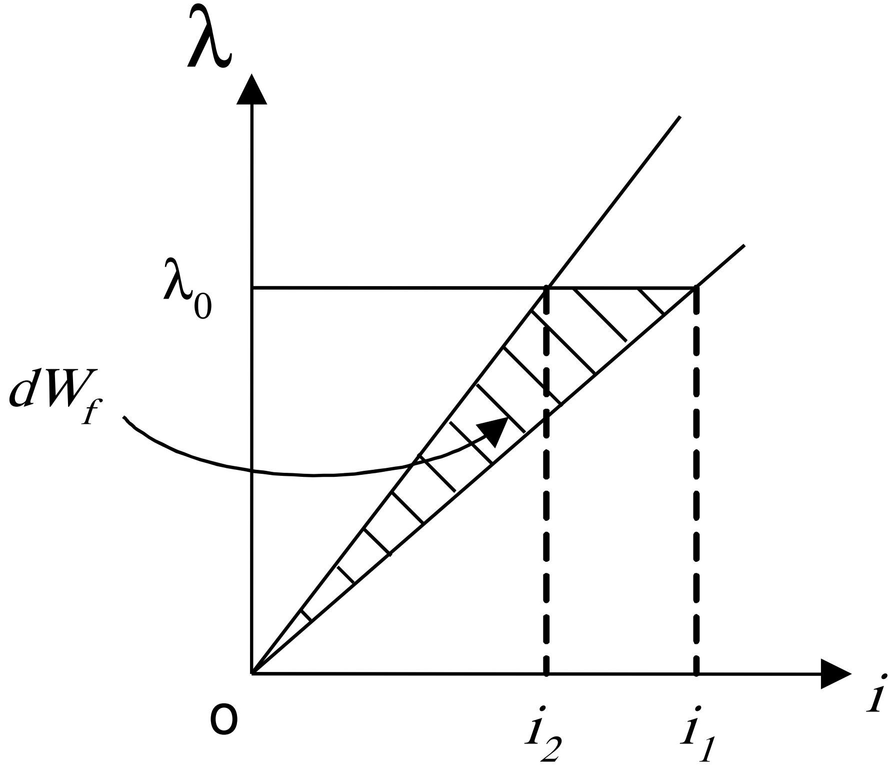


Fig. 3.8 Flux linkage is fixed

Mechanical Energy in a Linear System

Refer to Fig. 3.8, the change of field energy is

$$dW_f = \tfrac{1}{2}\lambda_o i_2 - \tfrac{1}{2}\lambda_o i_1$$

$$= \tfrac{1}{2}\lambda_o (i_2 - i_1) \tag{3.23}$$

Since λ does not change, no emf is induced in the coil , and

$$\therefore \qquad dW_e = 0 \tag{3.24}$$

$\therefore$ From Eq.3.6,

Mechanical Energy in a Linear System

$$dW_e = dW_m + dW_f$$

$$\therefore -F_m dx = dW_f \tag{3.25}$$

$$\Rightarrow \quad -F_m dx = \tfrac{1}{2}\lambda_o (i_2 - i_1) \tag{3.26}$$

= the change of field energy

Eq. 3.26 can be written as,

$$\boxed{F_m = -\frac{\partial W_f}{\partial x}(\lambda, x) \Bigg|_{\lambda = \text{constant}}} \tag{3.27}$$

Since the electrical input energy is zero, the mechanical output energy has been supplied entirely by the coupling field.

Mechanical Energy in a Linear System

For a linear electromagnetic system,

$$\lambda = L(x)\, i \tag{3.28}$$

where

$L(x)$ = the inductance of the coil which dependent on length of the air gaps.

From Eqs. 3.11 and 3.28,

$$W_f = \int_0^{\lambda} i\,d\lambda = \int_0^{\lambda} \frac{\lambda}{L(x)}\,d\lambda = \frac{\lambda^2}{2L(x)} \tag{3.29}$$

$$= \frac{L(x)^2 i^2}{2L(x)} = \tfrac{1}{2} L(x) i^2 \tag{3.30}$$

$$W_f = W_f' = \tfrac{1}{2} L(x) i^2 \tag{3.31}$$

Mechanical Energy in a Linear System

From Eqs. 3.22 and 3.31,

$$F_m = \frac{\partial W_f'}{\partial x}(i, x) \Big|_{i = \text{constant}}$$

$$= \frac{\partial}{\partial x}\left(\tfrac{1}{2} L(x) i^2\right) \Big|_{i = \text{constant}}$$

$$= \frac{1}{2} i^2 \frac{dL(x)}{dx} \qquad \text{——} \ \text{(3.32)}$$

$$\Rightarrow \qquad F_m = \frac{1}{2} i^2 \frac{dL(x)}{dx} \qquad \text{——} \ \text{(3.33)}$$

Mechanical Energy in a Linear System

From Fig. 3.1 (for linear system),

$$
\begin{aligned}
Ni &= H_u 2g \\
&= \frac{B_u}{\mu_o} 2g
\end{aligned}
\tag{3.34}
$$

From Eq. 3.18

$$
\begin{aligned}
\boldsymbol{W_f} &= \text{volume of air gaps} \times \frac{B_u{}^2}{2\mu_o} \\
&= A_u 2g \times \frac{B_u{}^2}{2\mu_o}
\end{aligned}
\tag{3.35}
$$

where $\boldsymbol{A_u}$ = cross section area of air gap

From Fig. 3.1, it is seen that a positive displacement dx will correspond to a reduction dg in the air gap length. Thus,

$$dx = -\,dg \text{ m} \qquad ③⑥ \;(3.36)$$

From Eqs. 3.27, 3.35 and 3.36 yield,

$$F_m = \frac{\partial}{\partial g}\left(A_u 2g \times \frac{B_u^{\,2}}{2\mu_o} \right)$$

$$\Rightarrow \qquad F_m = 2A_u\,\frac{B_u^{\,2}}{2\mu_o} \qquad (3.37)$$

where

$2A_u$ = The total cross-section area of air gaps

Mechanical Energy in a Linear System

$\therefore$ The force per unit area of air gaps, f_m is

$$f_m = \frac{B_u^{\;2}}{2\mu_o} \qquad \text{N/m}^2 \tag{3.38}$$

Example 3.2

An electromagnet system is shown in Fig. 3.9.

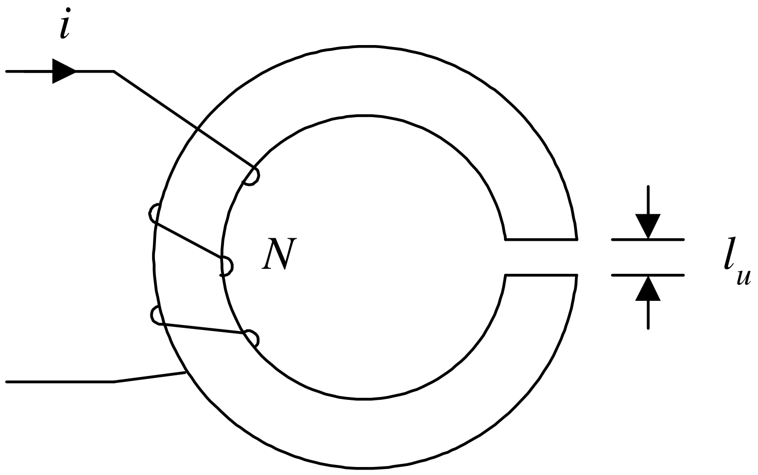


Fig. 3.9: linear system

Given that ***N*** = 600, ***i*** = 3 A, cross section area of air gap is 5 cm^2 and air gap length is 1.5 mm. By neglecting core reluctance, leakage flux and fringing effects, find:

(a) Force between the electromagnetic surfaces.

(b) Energy stored in the air gap.

Solution

(a) The total cross-section area of air gap = $\boldsymbol{A_u}$, Eq. 3.37 becomes,

$$F_m = A_u \frac{B_u^{\ 2}}{2\mu_o} \tag{3.39}$$

For linear system,

$$Ni = H_u l_u = \frac{B_u l_u}{\mu_o}$$

$$\Rightarrow \quad B_u = \frac{\mu_o Ni}{l_u} \tag{3.40}$$

Substitution from Eq. 3.40 in Eq. 3.39 yields

$$\therefore \quad F_m = \frac{A_u \mu_o N^2 i^2}{2 l_u^{\;2}}$$

$$= \frac{(5\times10^{-4})(4\pi\times10^{-7})(600)^2(3)^2}{2(1.5\times10^{-3})^2}$$

= 452.39 N

(b) Since the system is linear, the entire field energy is stored in the air gap,

$$W_f = \text{volume of air gap} \times \frac{B_u^{\,2}}{2\mu_o}$$

$$= l_u \times A_u \times \frac{B_u^{\,2}}{2\mu_o}$$

$$= \boldsymbol{l_u} \times \boldsymbol{F_m}$$

= (1.5 × 10-3) × 452.39 Nm

= 0.6789 Nm

= 0.6789 Joule

Example 3.3

Electromagnet system in Fig. 3.10 has cross-section area 25 cm^2. The coil has 350 turns and 5 ohm resistance. Magnetic core reluctance, fringing effects and leakage flux can be neglected. If the length of air gap is 4 mm and a 110 V DC supply is connected to the coil, find

(a) Stored field energy
(b) Lifting force

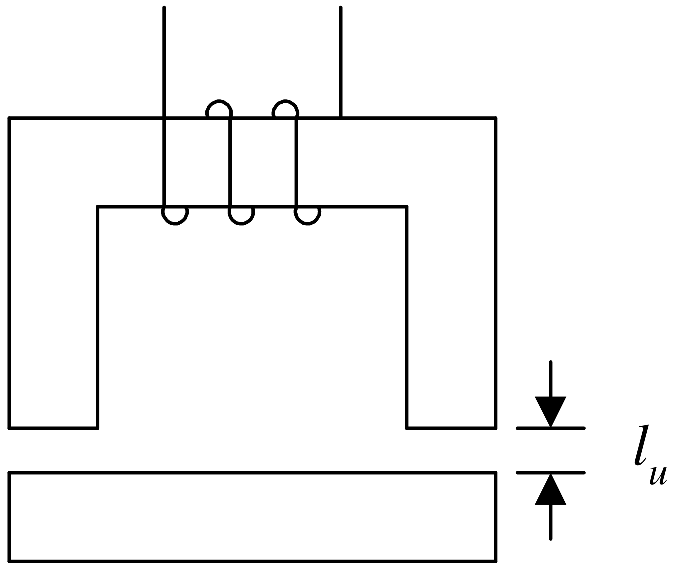


Fig. 3.10

Solution

Coil current,

$$i = \frac{110}{5} = 22 \qquad \text{A}$$

Since the electromagnet system is linear, core reluctance is neglected,

$$\therefore \qquad Ni = H_u l_u = \frac{B_u}{\mu_o} l_u$$

$$\therefore \qquad B_u = \frac{\mu_o Ni}{2l_u}$$

$$= \frac{(4\pi \times 10^{-7})(350)(22)}{2(4 \times 10^{-3})}$$

= 1.2095 Tesla

Field energy,

$$W_f = \text{volume of air gap} \times \frac{B_u^{\ 2}}{2\mu_o}$$

$$= 2\times(25\times10^{-4})\times(4\times10^{-3})\times\frac{1.2095^2}{2\times4\pi\times10^{-7}}$$

= 11.6413 Joule

(b) Applying Eq. 3.37 to obtain lifting force,

$$F_m = \text{the total area of air gaps} \times \frac{B_u^{\ 2}}{2\mu_o}$$

$$= 2\times(25\times10^{-4})\times\frac{1.2095^2}{2(4\pi\times10^{-7})}$$

= 2910.33 N

Mechanical Energy in a Saturable System

Figure 3.11 shows a diagram illustrating the λ-i characteristics for the actuator in Fig. 3.1 when the effect of the ferromagnetic material is taken into account. It is no longer a linear system due to saturation of the steel. W_f is smaller than coenergy W_f'.

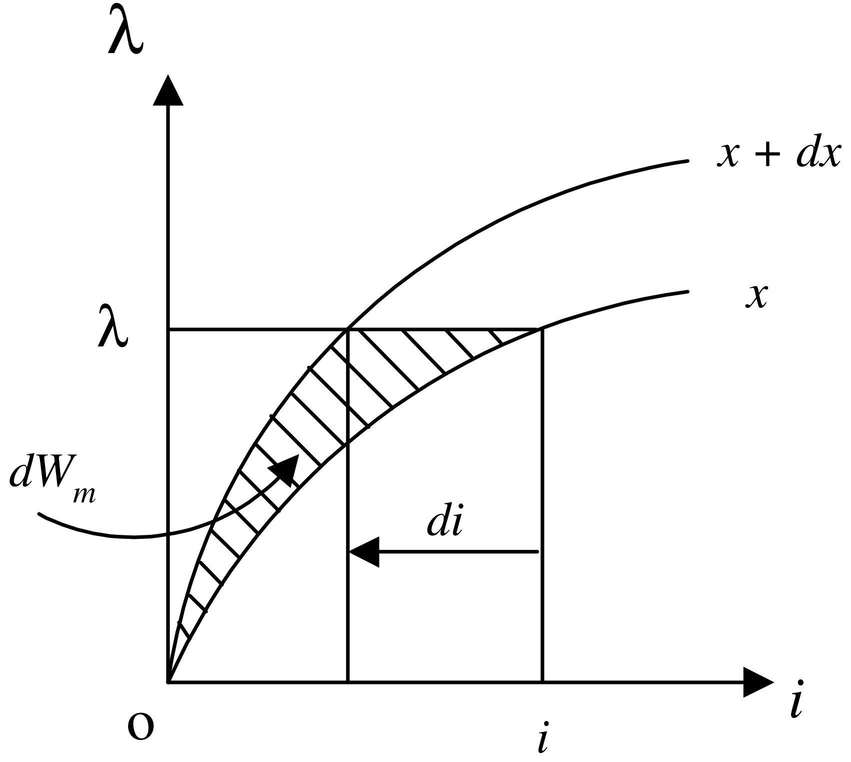


Fig. 3.11: At constant flux linkage.

Mechanical Energy in a Saturable System

and

$$F_m = \frac{dW_m}{dx} = -\frac{\partial W_f}{\partial x}(\lambda, x)\Bigg|_{\lambda = \text{constant}}$$

$$\Rightarrow \qquad F_m = -\frac{\partial W_f}{\partial x}(\lambda, x)\Bigg|_{\lambda = \text{constant}} \tag{3.43}$$

This corresponds to the expression for a linear system in Eq. 3.27

Mechanical Energy in a Saturable System

More usually, however, it is convenient to express λ as a function of ***x*** and ***i*** and to employ different approach. Figure 3.12 illustrates a differential movement of the operating point in the ***λ-i*** diagram corresponding to a differential displacement ***dx*** of the armature made at **low speed**; that is, at constant current. For this displacement, during flux linkage changes, the emf is not zero, and therefore ***dW_e*** is not zero.,

$$dW_e = \int eidt = \int_{\lambda_1}^{\lambda_2} id\lambda$$

$$= \text{area defg} \qquad \text{(3.44)}$$

$$dW_f = \text{area oef} - \text{area odg} \qquad \text{(3.45)}$$

Mechanical Energy in a Saturable System

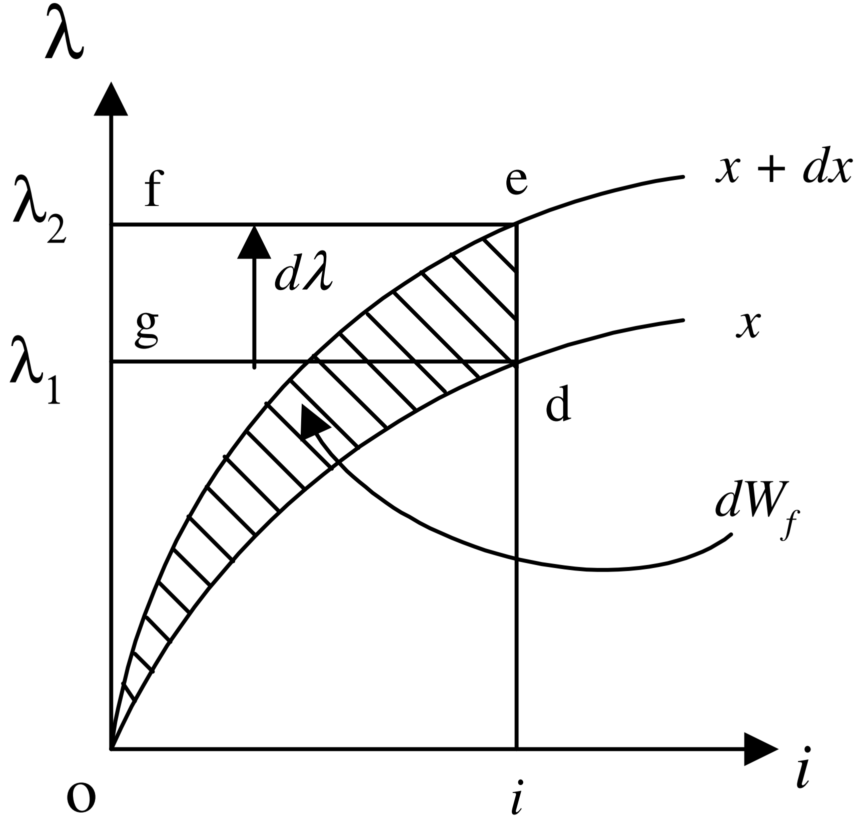


Fig. 3.12: At constant current

Mechanical Energy in a Saturable System

From Eq. 3.6,

$$\begin{aligned} dW_m &= dW_e - dW_f \\ &= \text{area defg} + \text{area odg} - \text{area oef} \\ &= \text{area ode} \end{aligned} \tag{3.46}$$

The differential mechanical energy associated with movement dx is given by the shaded area, it is equal to the increase of coenergy.

$$dW_m = dW_f'$$

$$\therefore F_m dx = dW_m = dW_f'$$

$$\Rightarrow \qquad F_m = \frac{\partial W_f'}{dx}(i, x)\Bigg|_{i = \text{constant}} \tag{3.47}$$

Mechanical Energy in a Saturable System

where

$$W_f'(i,x)=\int_0^i \lambda di \qquad \text{Joule} \tag{3.47}$$

W_f' is the function of i and x.

Also since $\lambda = N\Phi$, and $i = \mathscr{F}/N$, substitution in Eq. 3.47 yields the coenergy as a function of mmf and displacement

$$W_f'(\mathscr{F},x)=\int_0^{\mathscr{F}} \phi d\mathscr{F} \tag{3.48}$$

Mechanical Energy in a Saturable System

and

$$F_m = \frac{\partial W_f'}{\partial x}(\mathscr{F}, x)\Bigg|_{\mathscr{F} = \text{constant}} \qquad \text{③.49}$$

Example 3.4

The flux linkage and current relationship for an actuator can be expressed approximately by,

$$i = \left(\frac{\lambda g}{2} \right)^2$$

Between the limits $0 < i < 3$ A and $3 < g < 9$ cm. If the current is maintained at 2 A, what is the force on the armature for g = 4 cm?

Solution

The λ-*i* relationship is nonlinear, and thus the force must be determined using Eq. 3.11.

$$W_f = \int_0^{\lambda} i\,d\lambda = \int_0^{\lambda} \left(\frac{\lambda x}{2} \right)^2 d\lambda = \frac{x^2}{4}\frac{\lambda^3}{3} = \frac{1}{12} x^2 \lambda^3$$

From Eq. 3.43,

$$F_m = -\frac{\partial W_f}{\partial x}(\lambda, x) \Bigg|_{\lambda = \text{constant}}$$

$$= -\frac{\lambda^3 2x}{12}$$

For x = 0.04 m and i = 2 A,

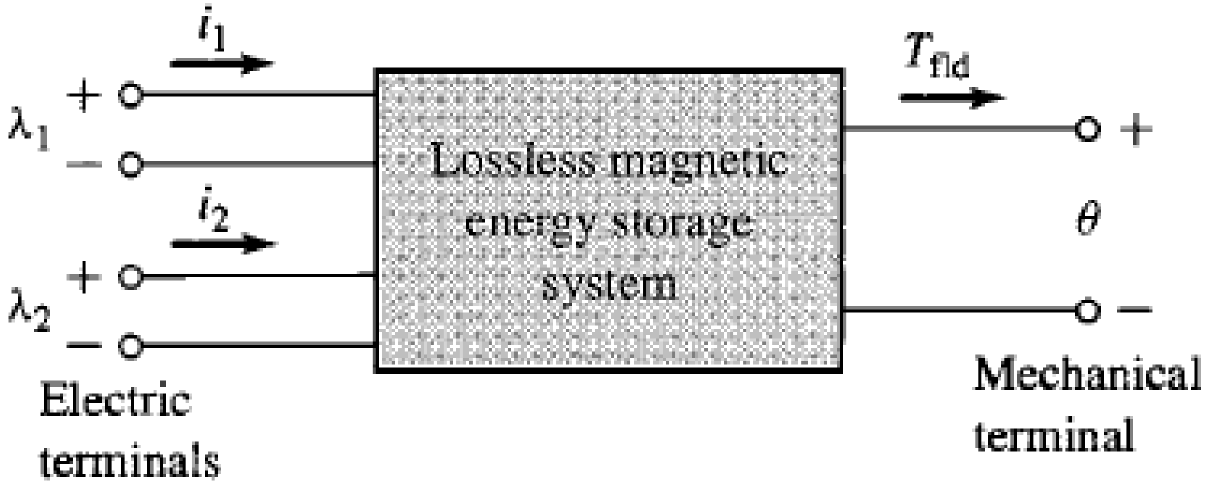
$$\lambda = \frac{2i^{\frac{1}{2}}}{x} = \frac{2 \times 2^{\frac{1}{2}}}{0.04} = 70.71$$ Wb-turn

$$\therefore \qquad F_m = -\frac{70.71^3 \times 2 \times 0.04}{12}$$

$$= -2356.95 \text{ N}$$

Multi-excited systems

- *When the electric system contains more than one circuit, then it is known as multi-excited system. Multi-excited systems are treated as singly excited systems in that energy and co-energy can be used to obtain the developed force or torque. Field energy and co-energy are found by taking the energy supplied by all circuits.*



$$T_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta)}{\partial \theta} \right|_{i_1, i_2} \qquad T_{\text{fld}} = - \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \theta} \right|_{\lambda_1, \lambda_2}$$

$$W'_{\text{fld}}(i_{1_0}, i_{2_0}, \theta_0) = \int_0^{i_{2_0}} \lambda_2(i_1 = 0, i_2, \theta = \theta_0)\, di_2 + \int_0^{i_{1_0}} \lambda_1(i_1, i_2 = i_{2_0}, \theta = \theta_0)\, di_1$$

$$W'_{\text{fld}}(i_1, i_2, \theta) = \frac{1}{2} L_{11}(\theta) i_1^2 + \frac{1}{2} L_{22}(\theta) i_2^2 + L_{12}(\theta) i_1 i_2$$

$$\begin{aligned} T_{\text{fld}} &= \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, \theta)}{\partial \theta} \right|_{i_1, i_2} \\ &= \frac{i_1^2}{2} \frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2} \frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} \end{aligned}$$

$$f_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}(i_1, i_2, x)}{\partial x} \right|_{i_1, i_2} \qquad f_{\text{fld}} = - \left. \frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, x)}{\partial x} \right|_{\lambda_1, \lambda_2}$$

$$W'_{\mathrm{fld}}(i_1, i_2, x) = \frac{1}{2}L_{11}(x)i_1^2 + \frac{1}{2}L_{22}(x)i_2^2 + L_{12}(x)i_1 i_2$$

$$f_{\mathrm{fld}} = \frac{i_1^2}{2}\frac{dL_{11}(x)}{dx} + \frac{i_2^2}{2}\frac{dL_{22}(x)}{dx} + i_1 i_2 \frac{dL_{12}(x)}{dx}$$

EXAMPLE 3.7

In the system shown in Fig. 3.15, the inductances in henrys are given as $L_{11} = (3 + \cos 2\theta) \times 10^{-3}$; $L_{12} = 0.3 \cos\theta$; $L_{22} = 30 + 10 \cos 2\theta$. Find and plot the torque $T_{\mathrm{fld}}(\theta)$ for current $i_1 = 0.8$ A and $i_2 = 0.01$ A.

Solution

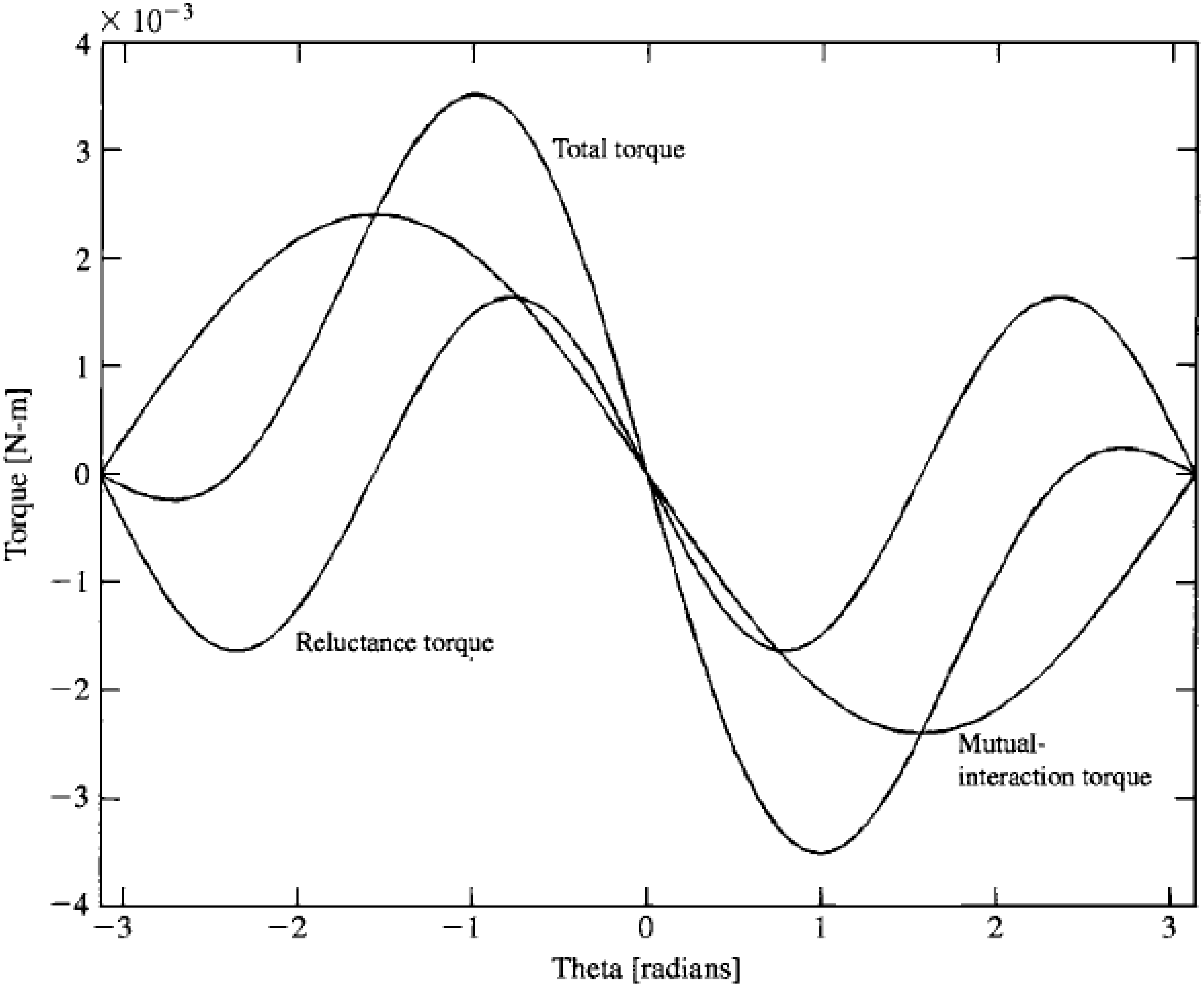
The torque can be determined from Eq. 3.71.

$$T_{\mathrm{fld}} = \frac{i_1^2}{2}\frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}$$

$$= \frac{i_1^2}{2}(-2 \times 10^{-3}) \sin 2\theta + \frac{i_2^2}{2}(-20 \sin 2\theta) - i_1 i_2 (0.3) \sin\theta$$

For $i_1 = 0.8$ A and $i_2 = 0.01$ A, the torque is

$$T_{\mathrm{fld}} = -1.64 \times 10^{-3} \sin 2\theta - 2.4 \times 10^{-3} \sin\theta$$


× 10^{−3}
4
3
2
1
0
−1
−2
−3
−4
Torque [N-m]
−3
−2
−1
0
1
2
3
Theta [radians]
Total torque
Reluctance torque
Mutual-
interaction torque

EXAMPLE

Find an expression for the torque of a symmetrical two-winding system whose inductances vary as

$$L_{11} = L_{22} = 0.8 + 0.27\cos 4\theta$$

$$L_{12} = 0.65\cos 2\theta$$

for the condition that $i_1 = -i_2 = 0.37$ A.

Solution

$$T_{\mathrm{fld}} = -0.296\sin(4\theta) + 0.178\sin(2\theta)$$

EXAMPLE

Two coils, one mounted on a stator and the other on a rotor, have self- and mutual inductances of

$$L_{11} = 3.5 \text{ mH} \quad L_{22} = 1.8 \text{ mH} \quad L_{12} = 2.1 \cos\theta \text{ mH}$$

where θ is the angle between the axes of the coils. The coils are connected in series and carry a current

$$i = \sqrt{2} I \sin \omega t$$

a. Derive an expression for the instantaneous torque T on the rotor as a function of the angular position θ.

b. Find an expression for the time-averaged torque T_{avg} as a function of θ.

c. Compute the numerical value of T_{avg} for $I = 10$ A and $\theta = 90°$.

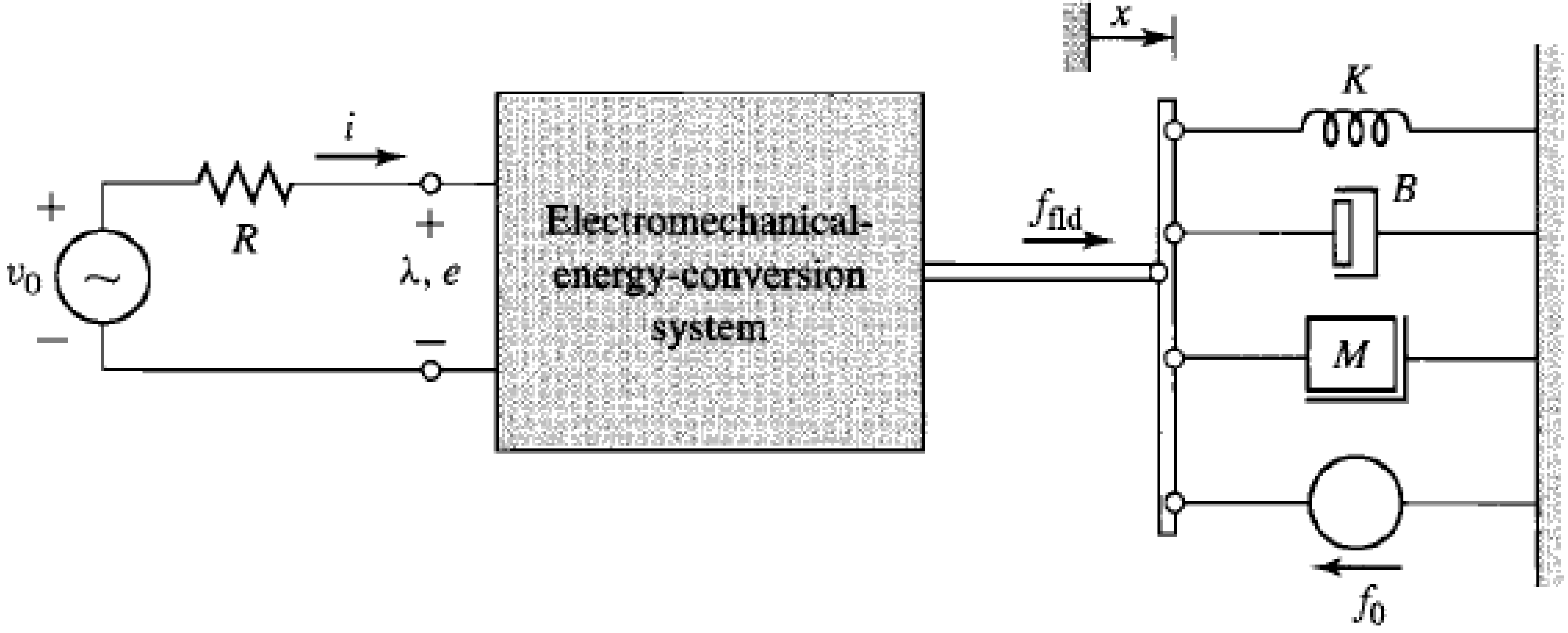
d. Sketch curves of T_{avg} versus θ for currents $I = 5, 7.07,$ and 10 A.

e. A helical restraining spring which tends to hold the rotor at $\theta = 90^\circ$ is now attached to the rotor. The restraining torque of the spring is proportional to the angular deflection from $\theta = 90^\circ$ and is $-0.1\ \mathrm{N \cdot m}$ when the rotor is turned to $\theta = 0^\circ$. Show on the curves of part (d) how you could find the angular position of the rotor-plus-spring combination for coil currents $I = 5, 7.07$, and 10 A. From your curves, estimate the rotor angle for each of these currents.

(Note that this problem illustrates the principles of the dynamometer-type ac ammeter.)

DYNAMIC EQUATIONS

The model of a simple electromechancal system shown in Fig. 3.23 shows the basic system components, the details of which may vary from system to system. The system shown consists of three parts: an external electric system, the electromechanical-energy-conversion system, and an external mechanical system. The electric system is represented by a voltage source v_0 and resistance R; the source could alternatively be represented by a current source and a parallel conductance G.



The electric equation for this model is

$$v_0 = iR + \frac{d\lambda}{dt}$$

$$v_0 = iR + L(x)\frac{di}{dt} + i\frac{dL(x)}{dx}\frac{dx}{dt}$$

For a multiply-excited system, electric equations corresponding to Eq. 3.87 are written for each input pair. If the expressions for the λ's are to be expanded in terms of inductances, as in Eq. 3.88, both self- and mutual-inductance terms will be required.

The mechanical system of Fig. 3.23 includes the representation for a spring (spring constant K), a damper (damping constant B), a mass M, and an external mechanical excitation force f_0. Here, as for the electrical system, the damper represents the losses both of the external mechanical system as well as any mechanical losses of the electromechanical-energy-conversion system.

The x-directed forces and displacement x are related as follows:

Spring:

$$f_K = -K(x - x_0)$$

Damper:

$$f_D = -B\frac{dx}{dt}$$

Mass:

$$f_M = -M\frac{d^2x}{dt^2}$$

where x_0 is the value of x with the spring normally unstretched. Force equilibrium thus requires that

$$f_{\text{fld}} + f_{\text{K}} + f_{\text{D}} + f_{\text{M}} - f_0 = f_{\text{fld}} - K(x - x_0) - B\frac{dx}{dt} - M\frac{d^2x}{dt^2} - f_0 = 0$$

Combining

$$v_0(t) = iR + L(x)\frac{di}{dt} + i\frac{dL(x)}{dx}$$

$$f_0(t) = -M\frac{d^2x}{dt^2} - B\frac{dx}{dt} - K(x - x_0) + f_{\text{fld}}(x, i)$$

There is no general analytical solution for these differential equations; they are nonlinear, involving products and powers of the variables x and i and their derivatives. They can be solved using computer-based numerical integration techniques.